Dimensional Analysis Introduction

Dimensional analysis is also known as unit conversion, factor analysis, factor label, factor unit system, unit analysis, DA, and unit multipliers. You have a great advantage once you understand dimensional analysis. You only need to know one definition about any new unit you are introduced to in order to use it to convert that unit in any problem. Dimensional analysis is a tool used in obtaining most science math solutions.

Dimensional analysis is a technique that is used to change any unit(s) from one to another. There are three basic concepts that are used in unit conversion:

1. We can **multiply or divide** any quantity **by one**, and the result will still be equal to the original quantity.

$$\text{Ex.} \quad 6 \ (\ \mathbf{1} \ ) = 6$$

2. The numeral **one (1)** can be written in many different ways when written in a fraction form.

$$\text{Ex.} \quad \mathbf{1} = \frac{1}{1} = \frac{2}{2} = \frac{3}{3} = \frac{4}{4} = \frac{10}{10} = \frac{11}{11} = \frac{feet}{feet} = \frac{miles}{miles} = etc.$$

As long as the numerator of the fraction equals the denominator of the fraction,(1 =-1, 2 = 2, 3 = 3, etc.), the fraction is equal to one (1). **Any definition** involving units **can** be made into a unit definition (conversion factor) **equal** to **one**.

$$\text{Ex.} \quad \mathbf{1} = \frac{12 \ inches}{1 \ foot} = \frac{2 \ feet}{24 \ inches} = etc.$$

Since any definition can be written as a fraction equal to one, this allows us to multiply by the conversion factor and not change the value of the original quantity but only what it looks like. In dimensional analysis we multiple by definitions (conversion factors) that equal one.

3. Reviewing fraction multiplication, we recall that we can first **cancel before multiplying**.

$$\text{Ex.} \quad \frac{2}{3} \ x \ \frac{6}{8} = \frac{1}{2}$$

3 and 6 can cancel to 1 and 2
2 and 8 can cancel to 1 and 4
2 and 4 can cancel or simplify to 1 and 2.

Basically any top can be canceled with any bottom when multiplying. **Units** act like numbers but **are separate from the number**. **Units** can also **cancel** tops and bottoms.

Example 1. 3 inches can be written as a fraction by putting it over 1

$$\frac{3 \ \ inches}{1}$$

We can take this fraction and multiply by one (1), the conversion factor, so by cancellation we get to a new unit, such as feet.

$$? \ ft \quad = \qquad \frac{3 \ \ inches}{1} \left(\frac{1 \ \ foot}{12 \ \ inches} \right) = \qquad \quad ft$$

When the units work out and we double check the definitions, 1 foot = 12 inches (it is easy to write 12 feet = 1 inch but a very noticeable error if we just take the time to read it), then we are ready to cancel the units first making sure the unit that remains is what you are seeking. Then look at all of the numbers in the problem and cancel if we can.

$$\frac{3 \ \ in}{1} \left(\frac{1 \ \ ft}{12 \ \ in} \right) = \frac{1}{4} \ ft \quad \textbf{or} \quad 3 \ \ inches = \frac{1}{4} \ foot$$

Example 2. If we were asked to find how many decades were in 5 centuries, we would start with 5 centuries and proceed to multiply by definitions that equal one (1) until the unit has changed from centuries into decades.

$$? \ decades \quad = \quad \frac{5 \ \ centuries}{1} \left(\frac{100 \ \ years}{1 \ \ century} \right) \left(\frac{1 \ decade}{10 \ \ years} \right) \quad = \qquad decade$$

Canceling the numbers we get the answer 50 decades. We now know

5 centuries = 50 decades.

Practice:

1. If we were asked to find how many seconds were in one year, we would start with 1 year and proceed to multiply by ones until the unit has changed from years into seconds. Try to set up this problem.

$$? \ \text{sec} = \frac{1 \ \ year}{1} \left(\frac{365 \ \ days}{1 \ \ year} \right) \left(\overline{\qquad\qquad} \right) \left(\overline{\qquad\qquad} \right) \left(\overline{\qquad\qquad} \right) = \qquad \text{sec}$$

2. Try to find out how many feet per second you are traveling when you go 55 miles per hour.

$$? \frac{ft}{\sec} = \frac{55 \ miles}{1 \ hr} \left(\frac{\qquad}{\qquad}\right) \left(\frac{\qquad}{\qquad}\right) \left(\frac{\qquad}{\qquad}\right) = \qquad \frac{ft}{\sec}$$

You start with miles on top and end with feet on top. Think about what you need to do to change that. You have hours on the bottom and want to end with seconds on the bottom. Think about what you need to do to change that.

Dimensional analysis seems impossible to learn without some practice. However, many students can't believe how easy this process is after they have had some practice with it. They feel like they are doing something "illegal" when they just conveniently put an unwanted unit where it will cancel out.

Click here for **answers** to the practice problems.

Click here for the **steps to use** in a dimensional analysis problem.